VP160 Recitation Class

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Lagrangian Mechanics

Momentum

Collisions

Center of Mass

Rocket propulsion

Degrees of freedom

The number of independent generalized coordinates needed to uniquely describe position of a particle. In general:

$$f = 3N - m$$

Lagrangian

The most significant quantity in Lagrangian Mechanics

$$L = K - U$$

Euler-Lagrange Equation

For $i = 1, 2, ..., f$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Defination

$$\overline{p} = m\overline{v}$$

Rewrite Newton's second law

$$\overline{F} = \frac{d\overline{p}}{dt}$$

Conservation of Momentum

1. If the sum of all external forces on the system is equal to zero, then the total momentum of the system is constant.
2. The total momentum of a system can only be changed by external forces.

Elastic Collisions

Only internal forces act, so both energy and momentum are conserved.

1. Equation 1: Conservation of energy;
2. Equation 2: Conservation of momentum;

Simple methods?

Inelastic Collisions

Only momentum is conserved.

1. Equation: Conservation of momentum;

Center of Mass

$$r_c = \frac{\sum m_i r_i}{\sum m_i}$$

$$r_c = \frac{\int r_i dm}{\int dm}$$

Pappus Law

$$V = 2\pi g S$$

Small Quiz

1. Find the center of mass of a half circle.
2. Find the center of mass of a half disk.
3. Find the center of mass of a half ball.
4. Find the volume of a doughnut.

Rocket Propulsion

$$mv + Fdt = (m + dm)(v + dv) - dmu$$
$$Fdt = mdv + dm(v - u)$$

General Equation

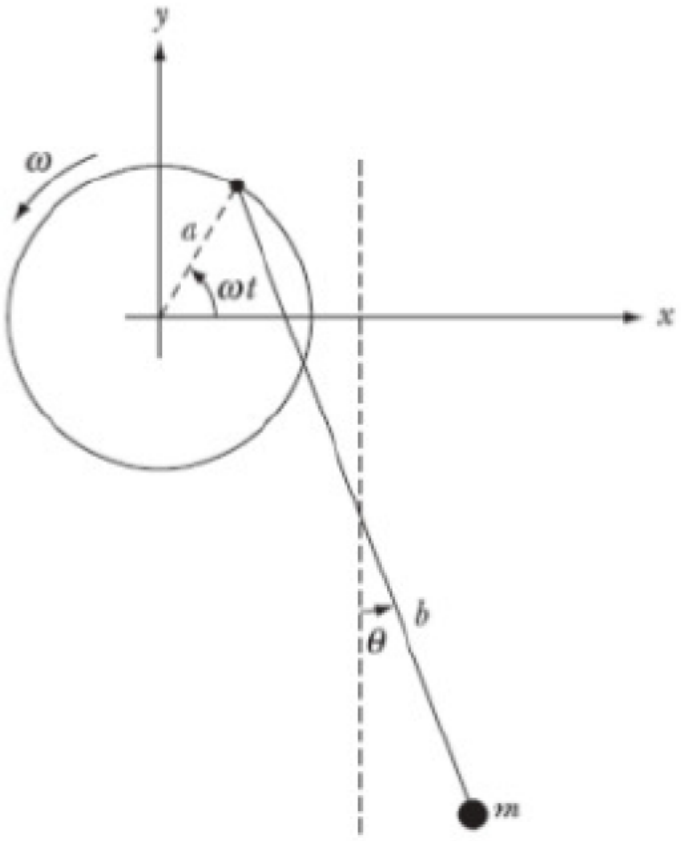
$$m\frac{dv}{dt} = (u - v)\frac{dm}{dt} + F$$

Reminder

What FoR are we looking at?

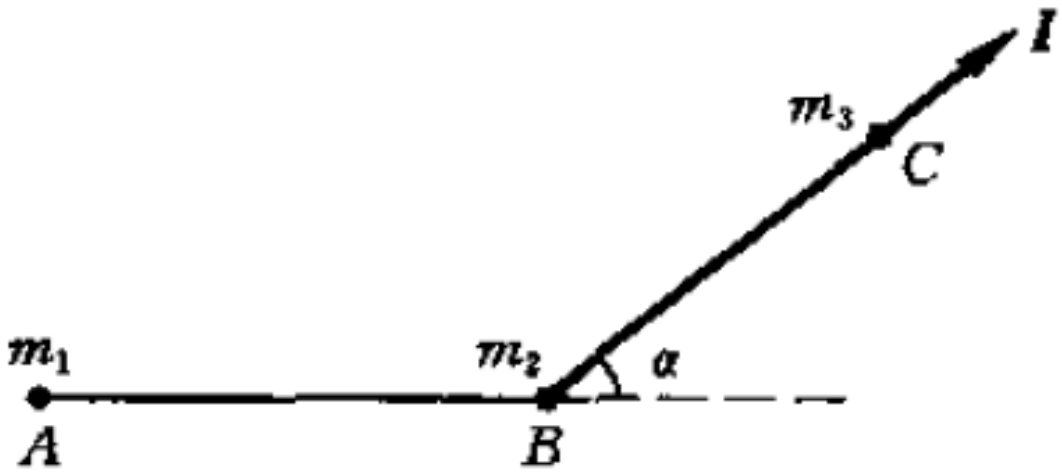
Exercise 1

A simple pendulum of length b and mass m moves attached to a massless rim of radius a rotating with constant angular velocity ω. Find the Lagrangian.



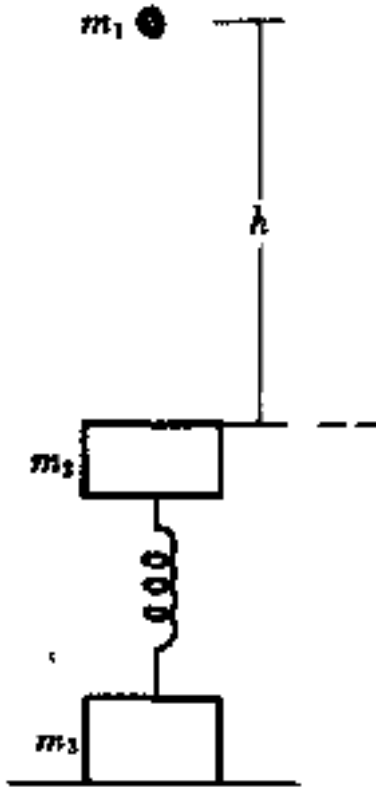
Exercise 2

Assume m_1, m_2, m_3, α is known. Apply I on C, what is the velocity of A at this instant?



Exercise 3

Assume m_1, m_2, m_3, k is known. Release m_1, the collision between m_1 and m_2 is completely inelastic. Fine h so that m_3 can just leave the ground.



Exercise 4

A rope with length l and mass m is placed vertically. At the beginning, the lower end of the rope just touches the ground. Release the rope, find the support force of the ground with respect to x.